Research on Mathematical Derivative Knowledge in Physical Extreme Value Problem

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Abstract: In the analysis of physical test questions, it is generally necessary to study the problems of increase, decrease, instantaneous and extreme values of physical quantities, but they cannot be solved smoothly according to elementary mathematics such as inequalities, quadratic functions and graphical analysis. At present, the knowledge of derivative is added to the content of mathematics teaching in the third year of high school, and this knowledge point has become a must-test point for the college entrance examination. In the high school physics review stage, the mathematical derivative knowledge can be used to solve the above problems. On the one hand, the derivative knowledge can be deepened and consolidated, and the physical problem-solving ideas can be broadened. The application of derivatives to the solution of physical extremum problems can help high school students develop divergent thinking and innovative consciousness, and develop students' ability to apply mathematical knowledge to solve physical problems. To this end, the paper combines several examples to analyze the application of derivatives in the solution of physical extremum problems, giving students some enlightenment.

1. Research background

The high school physical extremum problem is a kind of comprehensive problem integrating mathematics and physics knowledge. It is generally solved by knowledge methods such as matching method, mean inequality method and trigonometric function (auxiliary angle formula). It is difficult to solve problems and has better ability requirements. High, and the process is cumbersome. If students are not proficient in the corresponding mathematical knowledge and methods, they often cause obstacles in solving problems and become difficult problems. If the derivative method is used, the problem-solving process can be simplified and the effect can be achieved with half the effort. The steps of using the derivative to find the extremum of the physical quantity: firstly, the physical function equation is established according to the physical law; the second is to solve the first derivative $f'(x)$ of the physical quantity to be independent variable; the third is to find the root of the physical quantity $f'(x)=0$ within the variation range of the independent variable; It is a sign that judges $f'(x)$'s left and right values at the root of the equation, and determines whether $f(x)$ takes a maximum value (left positive right negative) or a minimum value (left negative right positive) at this root. Or re-seeking the second derivative $f''(x)=0$ of the independent quantity of the independent variable, the second derivative is not zero, then the point is the extreme point, the second derivative is greater than zero, this point is the minimum value; the second derivative is less than zero This point is the maximum value. This paper tries to analyze the application of the derivative method in the physical extremum problem through concrete examples [1].

It is not easy to solve the extremum by trigonometric function transformation, quadratic function deformation, and inequality. It is not easy to make a standard form of extremum, but after using the physical formula and physical law to establish the physical function equation, the derivative method is used step by step. For the relatively easy to achieve extreme value, the physical meaning is also relatively clear.

2. The Infiltration of Mathematical Thoughts in Physics Teaching in Middle Schools

2.1 The relationship between mathematics and physics.

Mathematics is a discipline that studies concepts such as quantity, structure, change, space, and information. Physics is the discipline that studies the most general laws of matter movement and the basic structure of matter. Both mathematics and physics describe the general laws of nature. With the deepening of people's understanding of natural phenomena and natural laws, it is especially important to introduce mathematical concepts into physics [2]. Without the description of mathematics, it is difficult for physics to understand the natural world from quantitative change to qualitative change, and it is difficult to change from qualitative research to quantitative research. Therefore, the development of physics is inseparable from mathematics. The mathematical ideas commonly used in high school stage include function thought, equation thought, classification thought, combination of number and shape, inductive thought, ultimate thought and collective thought. This paper will introduce the application of the combination of number and shape, function idea and limit idea in physics in combination with the derivative.

3. Infiltration of Derivatives in Physics Teaching in Middle Schools

The idea of combining numbers and shapes

The biggest feature of the combination of numbers and shapes is intuitive and concise. The combination of numbers and shapes allows students to learn the abstract concept knowledge, connect new knowledge and old knowledge, convert numbers and concepts into images, and associate and migrate, which plays an important role in solving physical problems.

Function Thought

The functional idea is a kind of thinking strategy. We can use this idea to find the characteristics of mathematical problems, build mathematical models based on functions, and then conduct research. The function idea is widely used in middle school physics. For example, the relationship between voltage and current in a closed circuit is a function. In the kinematics of high school physics, the relationship between time and displacement of uniform linear motion is a quadratic function. And decomposition is related to trigonometric functions.

Extreme thought

The ultimate idea is the basic idea of calculus. A series of important concepts in mathematical analysis, such as the continuity of functions, derivatives and definite integrals, are defined by means of limits. The general steps for solving the problem of limit thinking are as follows: 1. If you want to find the mathematical quantity solved in the problem setting, first try to conceive a variable related to it, and confirm that the result of the variable passing through the infinite process is the unknown quantity sought; Use the limit calculation to get this result.

4. Case study

4.1 Using the quadratic equation to find the root discriminant method

In the problem of finding the physical extremum whose number of equations is less than the unknown number, if the system of equations is collimated to obtain a binary equation about a certain unknown quantity, the discriminant can be used according to whether there is a real solution to the physical quantity. Condition should be satisfied, the new relationship should be listed, and the problem that the unknown is more than the equation group cannot be solved [3].

As shown in Figure 1, a certain mass of ideal gas changes from state A to state B. Knowing $T_A = 300K$, what is the maximum temperature of the ideal gas during the state change?

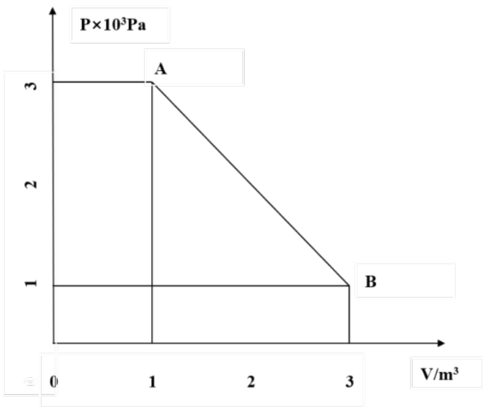

Figure 1. Gas change line diagram

Solution:

From the ideal gas state equation: $PV = nRT$, knowing $P_A = 3*10^5 P_a$, $T_A = 300K$, $V_A = 1m^3$
Solutions must:

$$\begin{aligned} &nR=10^3 J/K \\ &PV = 10^3 \left(J/K\right)*T \end{aligned} \tag{1}$$

From the image:

$$P=-10^5 \left(P_a/m^3\right)*V+4*10^5 \left(P_a\right) \tag{2}$$

Substituting equation (2) into equation (1): $-V^2+4V-0.01T=0$, volume V is a real number, so $\Delta \geq 0$, $16-4*0.01T \geq 0$. solves $T \leq 400K$, so the maximum temperature of the ideal gas in the state change is $T_m=400K$.

4.2 Using trigonometric functions

As shown in Fig. 2, two points of A,B are fixed at two points $2L$ from each other, and the charge amount is Q . At the point C on their mid-perpendicular line, a positive test charge (excluding gravity) with a quantity of q and a mass of m is released by rest. Try to check where the charge is moving to the maximum acceleration, what is the maximum acceleration? [4]

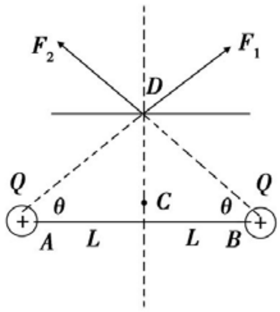

Figure 2. Charge diagram

Solution: Due to the symmetry, the force at the midpoint of AB is zero, and the resultant forces at other points on the perpendicular line in AB are along the direction of the vertical line. When q moves to point D on the mid-perpendicular line, it can be seen from the figure.

$$F_+=2F_1 sin\theta=\frac{2kQqsin\theta}{\left(L/cos\theta\right)^2} \tag{3}$$

$$a=\frac{F_{+}}{m}=\frac{2kQq}{mL^2}\left(sin\theta\text{-}sin^3\theta\right) \quad (4)$$

The following steps are solved by using the mean inequality, and the process is cumbersome. It is much more concise to solve with the derivative method:

Let $f\left(\theta\right)=sin\theta\text{-}sin^3\theta$ find the derivative $f'\left(\theta\right)=cos\theta\text{-}3sin2\theta cos\theta$ of $f\left(\theta\right)$. Let $f'\left(\theta\right)=0$, the solution: $sin\theta=\frac{\sqrt{3}}{3}$, $cos\theta=0$ (not the meaning of the question).

That is, when $\theta=arcsin\frac{\sqrt{3}}{3}$, $f\left(\theta\right)$ has a maximum value of $\frac{\sqrt{3}}{3}\text{-}\left(\frac{\sqrt{3}}{3}\right)^3=\frac{2\sqrt{3}}{9}$; the maximum value of acceleration is: $a_{max}=\frac{4\sqrt{3}kQq}{9mL^2}$.

5. Conclusion

Mathematical thinking has brought convenience to the study of physics and has been well used in practical teaching. In the ordinary education and teaching process, physics teachers should pay attention to infiltrating mathematics ideas, help students to go hand in hand in the process of learning physics and mathematics, and better combine the two courses to achieve better teaching results.

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